8. OPERATION LIMITS OF A PUMP-PIPELINE SYSTEM

8.1 DETERMINATION OF A REQUIRED MANOMETRIC PRESSURE IN A PUMP-PIPELINE SYSTEM

A lay out of a dredging pipeline, properties of transported solids and required mixture flow conditions (mixture velocity and density in a pipeline) determine a manometric pressure that must be produced by a dredge pump. The manometric pressure required to overcome the dredging-pipeline resistance is a pressure differential over a dredge pump, i.e. a differential between the pressure at the pump outlet to a discharge pipe and the pressure at the inlet to a pump connected with a suction pipe. If no geodetic height is assumed between the pump inlet and outlet

$$P_{\text{man}} = P_p - P_s + \frac{\rho_m (V_p^2 - V_s^2)}{2}$$  \hspace{1cm} (8.1)$$

For flow of mixture of density $\rho_m$ the absolute suction pressure at a pump inlet (Fig. 8.1)

$$P_s = P_{\text{atm}} + \rho_f g (h_{s,\text{pipe}} - h_{s,\text{pump}}) - \rho_f g H_{\text{totloss},s,m} - \frac{\rho_f V_s^2}{2}$$ \hspace{1cm} (8.2)$$

and the absolute discharge pressure at a pump outlet (Fig. 8.1)

$$P_p = \rho_m g (h_{d,\text{pipe}} + h_{d,\text{pump}}) + \rho_f g H_{\text{totloss},d,m} + P_{\text{atm}} - \frac{\rho_f V_p^2}{2}$$ \hspace{1cm} (8.3)$$

Where:

- $P_s$ = absolute suction pressure at a pump inlet [Pa]
- $P_{\text{atm}}$ = absolute atmospheric pressure [Pa]
- $h_{s,\text{pipe}}$ = depth of a suction pipe inlet below a water level [m]
- $h_{s,\text{pump}}$ = depth of a pump inlet below a water level [m]
- $H_{\text{totloss},s,m}$ = total head lost due to friction in a suction pipe [m]
- $V_s$ = mean velocity of mixture in a suction pipe [m/s]

- $P_p$ = absolute discharge pressure at a pump outlet [Pa]
- $h_{d,\text{pipe}}$ = vertical distance between a water level and a discharge pipe outlet [m]
- $h_{d,\text{pump}}$ = depth of a pump outlet below a water level [m]
- $H_{\text{totloss},d,m}$ = total head lost due to friction in a discharge pipe [m]
- $P_{\text{atm}}$ = absolute atmospheric pressure [Pa]
- $V_p$ = mean velocity of mixture in a discharge pipe [m/s]
The Eqs. 8.1 – 8.3 give a relationship between the manometric pressure delivered by a pump to mixture and the velocity of mixture in a pipeline connected to the pump. This relationship is further dependent on solids size and concentration in a pipeline and to a pipeline lay-out. The relationship is used to optimise the production and the energy consumption of a pump-pipeline system during a dredging operation. A suitable range of a system operation is confined by limits arising from processes occurring in a dredging pipeline. An entire system does not work successfully if a dredge pump operates outside the operational limits.

In a pump-pipeline system the flow rate of mixture must be controlled to remain within a certain range suitable for a safe and economic operation. The flow-rate range has a lower limit given by the deposition-limit velocity and an upper limit given by the velocity at which pump starts to cavitate.

8.2 THE UPPER LIMIT FOR A SYSTEM OPERATION: VELOCITY AT THE INITIAL CAVITATION OF A PUMP

A cavitation phenomenon is associated with low absolute pressure in a liquid. A cavitation is a condition in a liquid in which the local pressure drops below the vapour pressure and vapour bubbles (cavities) are produced. Cavitation decreases considerably a pump efficiency and might be a reason of a damage of pump components (pitting and corrosion). A cavitating pump provides lower manometric head and thus the lower production of solids by a dredging pipeline. The pump cavitation must be avoided during a pump-pipeline system operation.
8.2.1 Criterion for non-cavitational operation of a system

A pump begins to cavitate, i.e. cavitation occurs at the suction inlet to a pump impeller, if the Net Positive Suction Head (NPSH) available to prevent pump cavitation is smaller than NPSH required by a pump to avoid cavitation.

The no cavitation condition for a certain pump-suction pipe combination is:

\[(NPSH)_r < (NPSH)_a\]

in which:

The available \((NPSH)_a\) is a total available energy head over the vapour pressure at the suction inlet to the pump during an operation at velocity \(V_m\) in a suction pipe of a certain geometry and configuration.

\[
(NPSH)_a = \frac{P_s - P_{vapour}}{\rho_v g} + \frac{V_m^2}{2g} + \frac{P_{atm}}{\rho_v g} + h_{s,pipe} - H_{totloss,s,m} - S_m (h_{s,pipe} - h_{s,pump})
\]

\[
= \frac{P_{vapour}}{\rho_v g}
\]

\[(8.4)\]

\((NPSH)_a\) Net Positive Suction Head Available [m]

\(P_{vapour}\) vapour pressure [Pa].

The vapour pressure of a pumped medium limits the minimum absolute pressure that can be theoretically reached at the suction side of a pump. At this pressure the liquid (water) is transformed into steam. The steam bubbles develop in a water flow, they enter the pump and deteriorate its efficiency. The vapour pressure is dependent on the temperature of a medium. For water the typical values are:

<table>
<thead>
<tr>
<th>Temperature T [°C]:</th>
<th>Vapour pressure (P_{vapour}) [kPa]:</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.18</td>
</tr>
<tr>
<td>20</td>
<td>2.27</td>
</tr>
</tbody>
</table>

The lay-out of a suction pipe and flow conditions in a pipe determine the absolute suction pressure available at the pump inlet.
Figure 8.2. Net Positive Suction Head Available on a suction inlet of a pump.

The required \( (\text{NPSH})_r \) is a minimum energy head a certain pump requires to prevent cavitation at its inlet. This is a head value at the incipient cavitation. The \( (\text{NPSH})_r-Q \) curve is a characteristic specific for each pump and it must be determined by tests. A design (dimensions, shape) and an operation (specific speed) of a pump decide the absolute suction pressure at the initial cavitation.

\[
(\text{NPSH})_r = \frac{P_{s,\text{min}} - P_{\text{vapour}}}{\rho_f g} + \frac{v_m^2}{2g} \tag{8.5}
\]

- \( (\text{NPSH})_r \) Net Positive Suction Head Required [m]
- \( P_{s,\text{min}} \) minimum absolute suction pressure without cavitation [Pa]
- \( P_{\text{vapour}} \) vapour pressure [Pa].

At the incipient cavitation the absolute suction pressure \( P_{s,\text{min}} \) at the pump inlet is equal to the difference between the atmospheric pressure \( P_{\text{atm}} \) and the so-called "decisive vacuum" (Dutch: maatgevend vacuum) \( (\text{Vac})_d \), i.e.

\[
(\text{Vac})_d = P_{\text{atm}} - P_{s,\text{min}} \tag{8.6}
\]

The decisive vacuum is the relative suction pressure that represents a threshold criterion for a non-cavitational operation of a certain pump.

If a pump starts to cavitate it looses its manometric head. The \( (\text{Vac})_d \) is defined as the vacuum at the flow rate for which the manometric head is 95 per cent of the non-cavitational manometric head at the same pump speed (r.p.m.). The \( (\text{Vac})_d \) is
related with the flow rate in a “decisive-vacuum curve” in a H-Q plot (see Fig. 8.3). The decisive-vacuum curve is determined by a cavitation test.

![Graph](image_url)

**Figure 8.3.** Decisive vacuum (Dutch: Maatgevend vacuum) curve of a pump.

A substitution of Eq. (8.5) to Eq. (8.6) and rearranging gives a relationship between the \((NPSH)_r\) and the decisive vacuum \((Vac)_d\)

\[
\frac{(Vac)_d}{\rho \gamma g} = -(NPSH)_r + \frac{P_{atm}}{\rho \gamma g} - \frac{P_{vapour}}{\rho \gamma g} + \frac{V_m^2}{2g}
\]

(8.7).

As follows from the relationship between the \((HPSH)_r\) and the decisive vacuum \((Vac)_d\) a cavitation test gives also the \((NPSH)_r-Q\) curve, i.e. the minimum NPSH as a function of capacity \(Q\).
An upper limit for the working range of a pump-pipeline system is given by points of intersection of a pump decisive vacuum curve and a set of vacuum curves of a suction pipe for various mixture densities. The vacuum curve of a suction pipe summarises the friction, geodetic and acceleration heads over an entire length of the suction pipe to the total vacuum head and relates this head with a pump capacity (see Fig. 8.4a, 8.4b and 8.4c). The total vacuum head, $\frac{\text{Vac}}{\rho_f g}$, is a difference between the total absolute suction pressure head and the atmospheric pressure head

$$\frac{\text{Vac}}{\rho_f g} = \frac{P_{\text{atm}} - P_s}{\rho_f g} = S_m \left( h_{s,\text{pipe}} - h_{s,\text{pump}} \right) - h_{s,\text{pipe}} + H_{\text{totloss,s,m}} + \frac{V_s^2}{2g} \quad (8.8)$$

- $\text{Vac}$: vacuum; the pressure relative to atmospheric $P_{\text{atm}}$ [Pa]
- $\rho_f$: density of liquid [kg/m$^3$]
- $g$: gravitational acceleration [m/s$^2$]
- $P_s$: absolute suction pressure at a pump inlet [Pa]
- $P_{\text{atm}}$: absolute atmospheric pressure [Pa]
- $S_m$: relative density of mixture ($\rho_m/\rho_f$) [-]
- $h_{s,\text{pipe}}$: depth of a suction pipe inlet below a water level [m]
- $h_{s,\text{pump}}$: depth of a pump inlet below a water level [m]
- $H_{\text{totloss,s,m}}$: total head lost due to friction in a suction pipe [m]
- $V_s$: mean velocity of mixture in a suction pipe [m/s]

**Figure 8.3a.** Decisive vacuum curve and vacuum curves of a suction pipe for flow of mixture of various densities (schematic).
Figure 8.3b. Decisive vacuum curve and vacuum curves of a suction pipe transporting mixture of various densities from the depth 9 meter (after v.d.Berg, 1998).

Figure 8.3c. Decisive vacuum curve and vacuum curves of a suction pipe transporting mixture of various densities from the depth 18 meter (after v.d. Berg, 1998).

Table 8.1. Points of intersection between decisive vacuum curve and vacuum curves for different mixture densities; the intersection points determine the maximum production of solids attainable for given mixture density in a pump-pipeline system lifting mixture from a certain depth (see Fig. 9.1 in Chapter 9).

<table>
<thead>
<tr>
<th>Density in kg/m³</th>
<th>Qₘ m³/s</th>
<th>Qₘ m³/h</th>
<th>Qₘ m³/s</th>
<th>Qₘ m³/h</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>0</td>
<td>2,840</td>
<td>0</td>
<td>2,840</td>
</tr>
<tr>
<td>1050</td>
<td>2,659</td>
<td>479</td>
<td>2,756</td>
<td>496</td>
</tr>
<tr>
<td>1100</td>
<td>2,491</td>
<td>897</td>
<td>2,644</td>
<td>952</td>
</tr>
<tr>
<td>1150</td>
<td>2,309</td>
<td>1,247</td>
<td>2,538</td>
<td>1,370</td>
</tr>
<tr>
<td>1200</td>
<td>2,105</td>
<td>1,515</td>
<td>2,422</td>
<td>1,744</td>
</tr>
<tr>
<td>1250</td>
<td>1,891</td>
<td>1,702</td>
<td>2,320</td>
<td>2,088</td>
</tr>
<tr>
<td>1300</td>
<td>1,627</td>
<td>1,757</td>
<td>2,204</td>
<td>2,380</td>
</tr>
<tr>
<td>1350</td>
<td>1,273</td>
<td>1,604</td>
<td>2,089</td>
<td>2,632</td>
</tr>
<tr>
<td>1380</td>
<td>0,864</td>
<td>1,182</td>
<td>2,000</td>
<td>2,725</td>
</tr>
<tr>
<td>1400</td>
<td></td>
<td></td>
<td>1,956</td>
<td>2,816</td>
</tr>
<tr>
<td>1450</td>
<td></td>
<td></td>
<td>1,844</td>
<td>2,987</td>
</tr>
<tr>
<td>1500</td>
<td></td>
<td></td>
<td>1,693</td>
<td>3,047</td>
</tr>
<tr>
<td>1550</td>
<td></td>
<td></td>
<td>1,511</td>
<td>2,992</td>
</tr>
<tr>
<td>1600</td>
<td></td>
<td></td>
<td>1,288</td>
<td>2,782</td>
</tr>
<tr>
<td>1633</td>
<td></td>
<td></td>
<td>1,000</td>
<td>2,279</td>
</tr>
</tbody>
</table>
8.2.2 How to avoid cavitation

Basically, cavitation is avoided if the absolute suction pressure of a pump is maintained above a certain critical value. An analysis of the above explained cavitation criterion leads to the following proposals:
- to reduce the static head that the pump must overcome, i.e. to put the pump as low as possible (see par. 8.5)
- to reduce the head lost due to flow friction, i.e. to minimise local losses and a suction pipe length
- to increase pressure by using a larger pipe at the suction inlet of a pump (see par. 8.4).

During an operation (if the position of a pump and a geometry of a suction pipeline can not be changed) friction losses can be reduced
- either by diminishing the mean mixture velocity in a pipeline
- or by reducing the mixture density in a suction pipeline.

8.3 THE LOWER LIMIT FOR A SYSTEM OPERATION: VELOCITY AT THE INITIAL STATIONARY BED IN A PIPELINE

It was shown in the previous paragraph that high head loss due to too high velocity of mixture in a dredging installation might cause cavitation in a dredge pump and thus a considerable reduction of production and even a damage of a pump. On the other hand too low velocity might cause unnecessarily high head losses due to friction too. Furthermore the too low velocity might cause a blockage of a pipeline.

8.3.1 Criterion for a deposit free operation of a system

If settling mixtures are transported a portion of solids occupies a granular bed at the bottom of a pipeline. The part of solids that occupies the bed is strongly dependent on the mixture velocity in a pipeline. Under the increasing velocity the thickness of the bed tends to diminish because still more particles tend to be suspended due to increasing turbulent intensity of a carrying liquid. However, if the velocity is decreasing instead of increasing the bed becomes thicker and at certain velocity, called the deposition-limit velocity (or critical velocity), the first particles in the bed stop their sliding over a pipeline wall. If velocity decreases further the entire bed stops and, under certain circumstances, dunes might be developed at the top of a stationary bed. The flow becomes instable and a pipeline might be blocked. This is more likely to happen in some “critical” parts of a pipeline as are bends, particularly those to vertical pipe sections. A danger of blockage increases if solids occupy a considerable part of a total pipeline volume.

Even if a blockage is not likely to happen due to relatively low concentration and/or an absence of critical pipeline parts during a dredging operation, it is worthwhile to watch out the deposition-limit value of the mean mixture velocity in a pipeline. A
presence of a stationary bed means that solids that are actually not transported occupy a part of a pipeline. A stationary bed reduces a pipeline discharge area and so tremendously increases the frictional losses. Frictional losses not far below the deposition-limit velocity might be much higher than losses at even very high mixture velocities. On the other hand, an operation at velocity only slightly above the deposition-limit value is economic since the frictional loss at this velocity is usually considerably lower than at the extremes of a velocity range. The effects of velocity on the frictional losses and the variation of deposition-limit velocity under the different mixture flow conditions were discussed to details in earlier chapters.

The deposition-limit velocity is for most dredging operations considered the lower limit for a range of operational velocity. The boundary given by this velocity can be plotted to the H-Q (or I_m-V_m) plot as a curve connecting deposition-limit velocity values for different solids concentrations in a mixture flow of certain material in a pipeline of a certain diameter (see Fig. 8.5).

Figure 8.5. Locus curve giving a velocity at an initial stationary bed.

8.3.2 How to avoid a stationary bed in a pipeline

If the pipeline is composed of sections of different pipe sizes, the mixture flow rate must be maintained at the level assuring a super-critical regime (V_m > V_{dl}) in the largest pipe section (the section of the largest pipe diameter). Consider that in the largest section the mixture velocity is the lowest (continuity equation) and moreover the deposition-limit value of the mixture velocity is the highest because V_{dl} tends to grow with pipe diameter.

If the solids concentration fluctuates along a pipeline, the mixture flow rate must be maintained at the level assuring a super-critical regime in the section of an extreme concentration. For a prediction, use the highest value of the deposition-limit velocity from the entire range of expected solids concentrations. V_{dl} is sensitive to solids
concentration, it is always better to be slightly conservative in a determination of the appropriate value.

If during a job a dredging pipeline is prolonged, the flow rate supplied by a dredge pump might become insufficient to assure a super-critical regime in a pipeline. Then two solutions must be considered:
- to pump mixture at much lower concentration; this will lead to lower frictional losses and thus higher flow rate that might be high enough to avoid a thick stationary bed in a pipeline
- to install a booster station; this increases a manometric head provided by pumps and increase a flow rate.

If coarser solids must be pumped than expected when a dredging installation was laid out, the flow rate supplied by a pump might become insufficient to assure a super-critical regime in a pipeline. Then again the above two solutions must be considered.

8.4 EFFECT OF PIPE DIAMETER ON OPERATION LIMITS

For a certain required flow rate of mixture a larger pipeline means lower mean velocity in comparison with a smaller pipeline. This means that there is a better chance to pump a mixture without a danger of pump cavitation if a suction pipe is larger. Furthermore, a pipe resistance decreases with an increasing pipe diameter. This has also a positive effect with regard to a pump cavitation limit. On the other hand a possibility that a stationary bed will be developed in a pipeline increases with an increasing pipeline diameter.

A suction pipe larger than a discharge pipe is installed in some dredging installations. The diameter of a suction pipe is chosen to be of about 50 mm larger than that of a discharge pipe if a system is designed for transportation of fast-settling mixtures (flows of coarse or heavy particles). An operation at the suction side of a dredging pipeline is usually limited by a pump cavitation. For a certain mixture flow rate the velocity in a suction pipe is low and this helps to avoid cavitation. This is more important than a presence of a stationary bed that may possibly occur in a short suction pipe. The presence of a stationary bed is more dangerous in a long discharge pipe and since a cavitation is very unlikely to occur in a discharge pipeline the deposition-limit velocity limits an operation in a discharge pipeline. It is useful to choose smaller pipe diameter (when compared to a suction pipe) to avoid the sub-critical regime of mixture flow. A higher frictional loss and a higher wear of a pipeline wall of course pay this.
8.5 EFFECT OF PUMP POSITION ON OPERATION LIMITS

If a pump is placed to a lower position within a pump-pipeline system a suction pipe becomes shorter. A geodetic height over which a mixture has to be lifted in a suction pipe becomes smaller. In a shorter suction pipe pressure loss due to flow friction over a suction pipe length is lower than that in a suction pipe of an original length. A vacuum curve for a shorter suction pipe shows lower vacuum value at a certain flow rate for mixture of certain density. Thus a cross point between a decisive vacuum curve and a vacuum curve for a certain mixture density is reached at higher capacity Q (compare Fig. 8.4c and Fig. 8.6). Since the total resistance (expressed by a vacuum curve) of a suction pipe is lower in a shorter suction pipe than in a pipe of an original length the margin occurs between a net positive suction pressure required and available at a pump inlet. Consequently, pipe vacuum curves of mixture density higher than is that for an original pipe still cross the decisive vacuum curve of a pump. The mixture of density higher than in an original pipe can be pumped before an upper limit of a pump-pipeline operation is reached (compare Tab. 8.1 and Tab. 8.2). This means a considerable improvement of production. Therefore a submerged pump (a pump placed on a inclined pipe below a water level) is often used on dredging installations.

![Figure 8.6.](image)

**Figure 8.6.** Decisive vacuum curve and vacuum curves of a suction pipe transporting mixture of various densities from the depth 18 meter using a pump positioned 5 meter below the water level (after v.d. Berg, 1998).

**Table 8.2.** Points of intersection between decisive vacuum curve and vacuum curves for different mixture densities as shown on Fig. 8.6; the intersection points determine production of solids at conditions in a pump-pipeline system with a pump 5 meter below a water level (see Fig. 9.5 in Chapter 9).
8.6 OPERATION LIMITS ON A H-Q DIAGRAM OF A PIPELINE

Fig. 8.7 shows a working range of a dredge pump that pumps, with a constant pump speed, a mixture of a constant density through a discharge pipeline of variable length. The maximum length of the pipeline is limited by the deposition-limit velocity. If the pipeline would be longer the pressure delivered by the pump would not be enough to maintain the mean velocity of mixture in the discharge pipeline above the deposition-limit threshold. The minimum length of the discharge pipeline is limited by the decisive vacuum of a pump. If the pipeline would be shorter, the high mean velocity would cause so high frictional pressure losses in a suction pipe that cavitation would occur in a suction side of the pump.

Figure 8.7. Working range of a dredge pump in a pump-pipeline system. The deposition-limit (critical) velocity in a pipeline and the decisive vacuum of the pump limits the working range.
8.7 RECOMMENDED LITERATURE


CASE STUDY 8.1

For this Case study the same dredging installation and the same mixture flow conditions are considered as in Case study 7.1.

A deep dredge has a centrifugal pump on board. The heart of the pump is on the same geodetic height as the water level. The suction and the discharge pipes are mounted to the pump at the pump-heard level. The suction pipe of the dredge is vertical and the discharge pipe is horizontal. Both pipes have a diameter 500 mm. The dredge pump pumps the 0.2-mm sand from the bottom of the waterway that is 7 meter below the water level (thus the dredging depth is 7 meter). The density of a pumped sand-water mixture is 1400 kg/m$^3$. The discharge pipe is 750 meter long. The pump-pipeline installation is supposed to keep the production at 700 cubic meter of sand per hour.

1. Determine whether for the above described conditions the mean velocity through a pipeline high enough is to avoid a stationary deposit in the pipeline.
2. Determine whether for the above described conditions the pressure at the suction mouth of the pump is high enough to avoid cavitation. The minimum pressure for the non-cavitational operation is considered 3 x 10$^4$ Pa.

For the calculation consider the friction coefficient of the suction/discharge pipes $\lambda_f = 0.011$. The following minor losses must be considered:
- the inlet to the suction pipe: $\xi = 0.5$,
- the 90-deg bend in suction pipe: $\xi = 0.1$,
- the flanges in the suction pipe: $\xi = 0.05$,
- the flanges in the discharge pipe: $\xi = 0.25$,
- the outlet from the discharge pipe: $\xi = 1.0$.

Additional inputs:
\[ \rho_f = 1000 \text{ kg/m}^3 \]
\[ \rho_s = 2650 \text{ kg/m}^3 \]

Inputs:
\[ \Delta h_{\text{depth}} = 7 \text{ m} \]
\[ L_{\text{hor}} = 750 \text{ m} \]
\[ D = 500 \text{ mm} \]
\[ d_{50} = 0.20 \text{ mm} \]
\[ \rho_s = 2650 \text{ kg/m}^3, \quad \rho_f = 1000 \text{ kg/m}^3, \quad \rho_m = 1400 \text{ kg/m}^3 \]
\[ \lambda_f = 0.011, \quad \Sigma \xi = 1.9 \]
\[ Q_s = 700 \text{ m}^3/\text{hour} = 0.194 \text{ m}^3/\text{s} \]

Remark: To make a calculation simpler the effect of a pipeline roughness on frictional losses in a pipeline is considered to be represented by a constant value of the frictional coefficient $\lambda_f$, i.e. independent of variation of mean mixture velocity.
1. **Comparison of the actual velocity with the deposition-limit velocity**

Mean velocity of mixture in a pipeline, $V_m$:

$$C_{vd} = \frac{\rho_m - \rho_f}{\rho_s - \rho_f} = \frac{1400 - 1000}{2650 - 1000} = 0.2424 \text{ [-]}.$$

$Q_m = \frac{Q_s}{C_{vd}} = 0.1944 = 0.802 \text{ m}^3/s$,

$$V_m = \frac{4Q_m}{\pi D^2} = \frac{4 \times 0.802}{3.1416 \times 0.5^2} = 4.085 \text{ m/s}.$$

Deposition-limit velocity:

$V_{sm} = 2.9 \text{ m/s}$ (the Wilson nomograph, Fig. 4.8)

$V_{crit} = 3.3 \text{ m/s}$ (the MTI nomograph, Fig. 4.6)

The actual average velocity in a horizontal pipeline behind the pump is higher than the deposition-limit velocity. There will be no stationary deposit at the bottom of the pipeline for velocity $4.09 \text{ m/s}$.

2. **Comparison of the actual suction pressure at the pump with the minimum pressure for non-cavitational operation**

Energy balance for the suction pipe (the Bernoulli equation):

$$P_{inlet} = P_{suct} + \Delta P_{static} + \Delta P_{totloss,m} + \rho_f \frac{V_m^2}{2}$$

in which

$$P_{inlet} = P_{atm} + \Delta h_{depth}\rho_f g,$$

$$\Delta P_{static} = \Delta h_{depth}\rho_m g,$$

$$\Delta P_{totloss,m} = \left(\frac{\lambda_f}{D} \frac{\Delta h_{depth}}{D} + \sum \xi_{suct}\right) \rho_m \frac{V_m^2}{2}.$$

$\Delta P_{static}$ the static pressure differential between the inlet and the outlet of the suction pipe;

$\Delta P_{totloss,m}$ the total pressure loss (both major and minor) over the length of a pipe;

$P_{suct}$ the absolute pressure at the outlet of the suction pipe;

$P_{atm}$ the atmospheric pressure.

$$P_{suct} = P_{atm} - \Delta h_{depth} (\rho_m - \rho_f) g - \left(\frac{\lambda_f}{D} \frac{\Delta h_{depth}}{D} + \sum \xi_{suct}\right) \rho_m \frac{V_m^2}{2} - \rho_f \frac{V_m^2}{2}.$$

$$P_{suct} = 10^5 \cdot 7(1400 - 1000) \cdot 9.81 \cdot \left(\frac{7}{0.5} + 0.65\right) \cdot 1400 \cdot \frac{4.09^2}{2} \cdot 1000 \cdot \frac{4.09^2}{2} = 54.7 \text{ kPa}.$$ 

The absolute pressure at the suction mouth of the pump is $54.7 \text{ kPa}$. This is higher than the minimum non-cavitation pressure $30 \text{ kPa}$. The pump will not cavitate.
CASE STUDY 8.2

In Case study 7.2 the maximum length was determined of a pipeline connected with a centrifugal pump operating at its maximum speed if mixture of density 1412.5 kg/m$^3$ composed of water and a 0.3 mm sand is transported. The flow rate of pumped mixture was determined for a pipeline of a maximum length. It is necessary to check whether this flow rate is attainable in a system, i.e. whether it lays within an operational range of a pump-pipeline system.

Determine the limits of an operational range of a pump-pipeline system (the system is defined in Case study 7.2) and check whether the flow rate for the pipeline of the maximum length lays within this range. Determine the range of pipeline lengths in which a pump can operate at the maximum speed (475 rpm) if density of pumped mixture is 1412.5 kg/m$^3$.

Solution:

A. The upper limit for a system operation:

CALCULATION:

a. Pump characteristics

The decisive-vacuum curve of the IHC pump can be approximated by the equation

\[
(Vac)_d = 94.99 - 3.64Q_m - 2.43Q_m^2 \quad [kPa] \quad (C8.1).
\]

b. Suction pipeline characteristics

The vacuum-curve equation (Eq. 8.8) for a suction pipeline of
\[
\Delta h_{s,pipe} = \Delta h_{depth} = 15 \text{ m} \\
\Delta h_{s,pump} = 0 \text{ m}
\]
gets a form of Eq. C7.13. This equation is solved for the following input values
\[
\omega = 45 \text{ deg} \\
L_{\text{horiz,suction}} = 2 \text{ m}
\]

Minor-loss coefficient: Suction pipeline: pipe entrance: $\xi = 0.4$

all bends, joints etc.: $\xi = 0.3$

Total value: $\Sigma \xi = 0.7$

\[
Vac = \Delta p_{\text{totalpipe,m}} = 0.28498Q_m^2 \left(2 + \frac{15}{\sin(45)}\right) + Q_m^{1.7} (S_m - 1)(0.67924x2 + 0.35774\frac{15}{\sin(45)}) + 12.97x0.7 S_m Q_m^2 + 9.81 (S_m - 1)x15 \quad [kPa] \quad (C8.2).
\]

For $S_m = 1.4125$
\[ \text{Vac} = \Delta p_{\text{totalpipe,m}} = 0.28498 Q_m^2 \left( 2 + \frac{15}{\sin(45)} \right) + Q_m^{-1.7} 0.4125(0.67924x2 + 0.35774 \frac{15}{\sin(45)}) + 12.97x0.7x1.4125Q_m^2 + 9.81x0.4125x15 \text{ [kPa]} \]

**Balance:**

\[ (\text{Vac})_d = \text{Vac} \]

(Eq. C8.1)  (Eq. C8.2)

determines the flow rate value (\(Q_{\text{upper}}\)) at the beginning of cavitation of a pump. This flow-rate value is the upper limit of an operational range of a pump-pipeline system pumping an aqueous mixture of 300-micron sand at mixture density 1412.5 kg/m³. Only operation at flow rates lower than this threshold value will be cavitation free.

**OUTPUT:**

For \(S_m = 1.4125\) the upper limit of a pump-pipeline operation is given by

\[ Q_{\text{upper}} = 1.115 \text{ m}^3/\text{s}. \]

**B. The lower limit for a system operation:**

The lower limit is given by the flow rate value (\(Q_{\text{lower}}\)) at the critical (deposition-limit) velocity.

**CALCULATION:**

The MTI correlation (Eq. 4.19) for the critical velocity gives

\[ V_{\text{crit}} = 1.7 \left( 5 - \frac{1}{\sqrt{0.3}} \right) \sqrt{0.5 \left( \frac{0.25}{0.25 + 0.1} \right) \frac{1}{6} \sqrt{\frac{2.65 - 1}{1.65}}} = 3.61 \text{ m/s}, \]

\[ Q_{\text{lower}} = V_{\text{crit}} A = V_{\text{crit}} \frac{\pi D^2}{4} = 2.92 \frac{\pi x 0.5^2}{4} = 0.709 \text{ m}^3/\text{s}. \]

**OUTPUT:**

For \(S_m = 1.4125\) the lower limit of a pump-pipeline operation is given by

\[ Q_{\text{lower}} = 0.709 \text{ m}^3/\text{s}. \]

**C. The range of lengths of an entire pipeline:**
Balance

\[ P_{\text{man},m} = \Delta P_{\text{total pipe},m} \]  
(Eq. C7.5)  
(Eq. C7.13)

- for the working point at \( Q_{\text{upper}} = 1.115 \text{ m}^3/\text{s} \) gives the length of an entire pipeline \( L = 610 \text{ meter} \). This is the minimal length for which the 1412.5 kg/m\(^3\) mixture can be pumped. If the pipeline becomes shorter, the flow rate tends to increase. This would cause cavitation at the inlet of a pump. The density of transported mixture in a short pipeline must be lowered to avoid cavitation;

- for the working point at \( Q_{\text{lower}} = 0.709 \text{ m}^3/\text{s} \) gives the length of an entire pipeline \( L = 1123 \text{ meter} \). This working point lays at the descending part of a pipeline resistance curve. An operation at this part of the curve should be avoided, since it is potentially instable and energy costly (see Fig. C7.4). The recommended minimum flow rate for pumping the 1412.5 kg/m\(^3\) mixture at the maximum speed of the pump is that the maximum length of a pipeline (see Case study 7): \( L_{\text{max}} = 975 \text{ meter} \), i.e. \( Q_{\text{minimum}} = 0.897 \text{ m}^3/\text{s} \).